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AN APPROXIMATE EXPRESSION FOR THE VALUE OF AN ASSURANCE, LIFE AGAINST LIFE.

To the Editor of the Assurance Magazine.

SIR,—The following short investigation may, perhaps, be useful to some of your readers. It will be observed that the resulting formula is much more simple than the ordinary one for the single premium for a contingent assurance; and it has been found to be sufficiently accurate for practical purposes.

PROBLEM.

To find the present value of £1 to be received at the end of the year in which a life aged x may fail, provided that such event happen during the lifetime of another, aged y; the chance of both dying in the same year being neglected.

$$\text{Let } \mathrm{S} = \text{value required;}$$

$$\text{then } \mathrm{S} = \Sigma v^n (p_{x\ n-1} - p_{x,n}) p_{y,n}$$

$$= \Sigma v^n (p_{x,n-1} p_{y,n} - p_{xy,n});$$

$$\text{but } \Sigma v^n p_{xy,n} = a_{xy}$$

$$\text{and } \Sigma v^n p_{x,n-1} p_{y,n} = \frac{l_x . l_{y+1}}{l_x . l_y} v + \frac{l_{x+1} . l_{y+2}}{l_x . l_y} v^2 + \frac{l_{x+2} . l_{y+3}}{l_x . l_y} v^3 + \text{\&c.}$$

$$= \frac{l_{y+1}}{l_y} v \left(1 + \frac{l_{x+1} . l_{y+2}}{l_x . l_{y+1}} v + \frac{l_{x+2} . l_{y+3}}{l_x . l_{y+1}} v^2 + \text{\&c.} \right)$$

$$= v p_y (1 + a_{x,y+1})$$

$$\therefore \mathrm{S} = v p_y (1 + a_{x,y+1}) - a_{xy}.$$

I am, Sir,

Yours truly,

ARTHUR H. BAILEY.

Equity and Law Life Office,
13*th March*, 1861.

INSTITUTE OF ACTUARIES.

PROCEEDINGS OF THE INSTITUTE.

First Ordinary Meeting, Session 1860-61.—*Monday,* 26*th November,* 1860.

CHARLES JELLICOE, President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

Messrs. Alexander Burnett, George Thomas Ruck, James Stark, Jun., and Mark Symons, duly nominated at the last ordinary meeting, were unanimously elected Associates of the Institute.

Mr. J. Hill Williams read a paper—"On the theory of probabilities," by Robert Campbell, Esq., M.A.

Thanks were voted to Mr. Williams and to Mr. Campbell, and the meeting adjourned to the 31st December, 1860.

Second Ordinary Meeting, Session 1860-61.—*Monday,* 31*st December,* 1860.

CHARLES JELLICOE, President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

The undermentioned gentlemen, duly nominated at the last ordinary meeting, were unanimously elected members of the Institute:—

Official Associate—Henry D. Davenport, Esq.

Associates.

Mr. R. Clarke.	Mr. C. R. Saunders.
„ W. R. D. Gilbert.	„ T. Y. Strachan.
„ A. W. Mackenzie.	„ C. J. Wilkins.

The Chairman announced that out of eleven candidates for the matriculation examination (1860), nine passed, in the following order of merit:—

	Marks.		Marks.
Mr. C. R. Saunders	440	Mr. James Stark	332
„ R. P. Hardy	364	„ C. J. Wilkins	293
„ J. R. Knowles	344	„ T. Y. Strachan	278
„ Fredk. Harper	335	„ Jas. Henderson	271
„ H. W. Manly	331		

Mr. Hodge then read a paper—"On the stability of results based on average calculations," by Robert Campbell, Esq., M.A.

Thanks were voted to Mr. Campbell and Mr. Hodge, and the meeting adjourned to Monday, 28th January, 1861.

Third Ordinary Meeting, Session 1860-61.—*Monday,* 28*th January,* 1861.

CHARLES JELLICOE, President, in the Chair.

The minutes of the last ordinary meeting were read and confirmed.

The Secretary announced various donations to the library.

The undermentioned gentlemen, duly nominated at the last ordinary meeting, were elected members of the Institute:—

Official Associate—Frank McGedy, Esq.

Associates.

Mr. F. Harper.	Mr. J. R. Knowles.
„ Jas. Henderson.	„ Henry Wm. Manly.

Mr. Tucker read a paper—"On the rates of premium required to provide for certain periodical returns to the assured."

Thanks having been voted to Mr. Tucker, the meeting adjourned to Monday, 25th February, 1861.

Fourth Ordinary Meeting, Session 1860-61.—*Monday,* 25*th February,* 1861.

CHARLES JELLICOE, President, in the Chair.

Read and confirmed the minutes of last ordinary meeting.

Various donations to the library were announced.

The undermentioned gentlemen, duly nominated at the last ordinary meeting, were elected members of the Institute, viz.:—

Fellow—W. S. B. Woolhouse, Esq.

Associates.

Mr. C. F. Haycraft. | Mr. S. J. Shrubb.

Mr. Sprague read a paper "On Mr. Gompertz's law of mortality."

Thanks having been given to Mr. Sprague, the meeting adjourned to Monday, the 25th March, 1861.

ANNUAL EXAMINATIONS.

The members of the Institute will, no doubt, recollect that early in the last year some alterations were suggested by the examiners in the syllabus published by the Institute; and that their suggestions being approved by the Council, the amended syllabus was adopted and ordered to be thereafter acted upon.

The changes then made have necessarily affected, to a certain extent, the character of the questions; and as it is obviously desirable that the way in which these changes have operated should be understood, we now publish the questions given on the last occasion for the second and third years' examinations — those for the first year being but very slightly influenced by them. It will be seen that the third year's questions are those mainly affected by the alterations.

SECOND YEAR'S EXAMINATION, 1860.

1. Assuming the formula for $\log_e(1+x)$, prove that

$$\log_e 3=1+\frac{1}{3}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{1}{4^2}+\frac{1}{7}\cdot\frac{1}{4^3}+\dots$$

$$\tfrac{1}{2}\log_e 10=\log_e 3+\frac{1}{19}+\frac{1}{3}\cdot\frac{1}{19^3}+\frac{1}{5}\cdot\frac{1}{19^5}+\dots$$

2. Explain what is meant by the modulus of a system of logarithms. By means of the formulæ in question, show that

$$\log_e 3 = 1{\cdot}098612$$
$$\log_e 10 = 2{\cdot}302585$$

Also show that the modulus of the common system of logarithms is ·434294.

3. When an event can happen in more ways than one, show that the probability of its happening is equal to the sum of the probabilities in respect of the different ways.

4. Determine (1) the probability that two persons now aged x and y respectively will both die in the nth year from the present time; (2) the probability that one or both of them will die in that year.

5. Find the present value of a sum of money due at the end of any time, assuming the operation of compound interest.

Prove that the value of £1 due at the end of $p+q$ years=value of £1 due in p years$\times$value of £1 due in q years.

6. Describe a practical method of calculating a table of the value of £1 due at the end of any number of years from 1 to 100; and show how the

results may be employed to calculate a table of the values of annuities certain.

Ex.—The value of £1 due at the end of 10 years, at 4 per cent., is ·67556417: deduce the first ten terms of the value of an annuity for any number of years.

7. Explain what is meant by a table of mortality, and state the different methods in which the facts embodied in it may be exhibited.

8. Give some account of the origin and relative merits of the tables of mortality known as the Carlisle, Equitable, Experience, and English Life Tables.

9. State accurately what is meant by the expectation (or mean duration) of life at any age according to a given table of mortality.

Prove the formula for calculating the chance of living a year at any age from the expectation—

$$p_x = \frac{e_x - \frac{1}{2}}{e_{x+1} + \frac{1}{2}}.$$

10. Prove that if $e_{x-1} = e_x = e_{x+1}$, then will $p_{x-1} = p_x$; but that if $p_{x-1} = p_x = p_{x+1}$, e_{x-1} will not be equal to e_x unless $e_{x+2} = \frac{1}{2} \cdot \frac{1+p_x}{1-p_x}$.

11. The value of an annuity on the life of a person of a given age is frequently supposed to be equal to that of an annuity certain for a number of years equal to the mean duration of life at that age. Explain why this is not the case, and state upon what hypothesis it would be true.

12. If B represents the present value of a benefit of £1 upon a given life (x), B_1 the same upon a life one year older ($x+1$), p the probability of a payment of B being received in the first year, and Π the probability of (x) surviving a year, prove that

$$\log B = \log v\,\Pi + \log\left(\frac{p}{\Pi} + B_1\right).$$

13. Describe the process of calculating a table of annuities by means of the formula $a_{m-1} = (1 + a_m)p_{m-1,1}.r$; and show that this formula is a particular case of the one in the last question.

14. Explain fully the construction and use of columns D, N, and M, in the columnar method of calculating the values of annuities and assurances; also state the superior advantages this method possesses over the old one.

15. Prove that the value of an annuity of £1 during the joint lives of x and y, and for t years afterwards, should x survive so long, is

$$a_x - \frac{D_{x+t}}{D_x}(a_{x+t} - a_{x+t.y}).$$

16. Give an expression for the annual premium for a contingent annuity to commence at the death of A and to continue as long as either B or C is living.

17. Find, in a convenient form for computation, the single and annual premiums for an annuity to commence at the death of y and continue payable during the remainder of x's life, but to be payable only if y dies within t years.

18. Prove that the single premium for an assurance payable on a life now aged n years attaining the age of $n+t$, or dying previously, may be

represented by the formula (Jones's notation), $\frac{1-ia_{n_{\overline{t-1}|}}}{1+i}$; and show that this formula is equivalent to $\frac{M_n - M_{n+t} + D_{n+t}}{D_n}$.

19. Give an expression for the single premium for an assurance on the life of A provided he die after B.

20. Prove that $A_{\overset{}{\underset{1}{(x.y)}}\overline{|}t} = r^t . p_{(x.y)t} . A_{\underset{1}{(x+t.y+t)}}$.

21. Determine the value of an assurance on a life of 30, payable at the age of 60 or previous death, commencing at £a, increased by £p at the end of 5 and 10 years respectively, and thereafter increasing by £q *per* annum.

22. Prove the formulæ for the value of a policy of £1—

$$(1)\ 1-\frac{1+a_{m+n}}{1+a_m},\quad (2)\ (P_{m+n}-P_m)(1+a_{m+n}),\quad (3)\ 1-(d+P_m)(1+a_{m+n}).$$

Show that the same forms apply when the policy is on the joint duration of two lives.

23. Find the value of an annuity on two successive lives, x and y, of which the second is to be nominated at the death of the first, and is supposed to be then y years of age. Also show that if I represent the value of a perpetuity of £1, the value of the annuity will be equal to $I-(1+I)A_xA_y$.

24. Find the single premium for an annuity to x after the death of y, with the condition that the premium is to be returned if x die before y.

25. A, aged x years, is entitled to the interest of £1 for life. If A die within t years, the interest is payable to B, or his representatives, till the expiration of the t years; when C, if living, is entitled to the capital. If, however, C, now aged y years, die either before A or within the t years, the capital reverts to B or his representatives. Determine the values of B's and of C's interests.

Third Year's Examination, 1860.

1. Describe Mr. Gompertz's method of graduating tables of mortality, and give his formula.

2. Describe the method proposed by Mr. Milne for that purpose.

3. Give a brief description of the other methods which have been devised with the like object.

4. Say which of these is, in your opinion, the most effective; and give your reason for thinking it so.

5. In what way do you consider that the surplus of an Assurance Company can be most equitably distributed amongst the assured?

6. When an assurance is effected by one person on the life of another, state the effect of an untrue statement by the life assured or the referees upon the validity of the policy.

7. Explain the difference in the constitution—(1) of Companies formed under the Act 7th and 8th of Victoria, c. 110; (2) of Companies formed before the passing of that Act, and having a special Act of Parliament; (3) of Companies so formed without a special Act; giving instances of each among the existing Companies.

8. How does this difference affect proceedings in the courts of law?

9. What qualifications are mainly necessary in a deed assigning a policy of assurance?

10. What legal remedy has an assignee, should a Company decline to pay under his policy?

11. Explain the process of forming a table of mortality from the burial registers of a stationary population, and the means of correcting for increase of population.

12. Point out the errors of the Northampton Table, and show how they arose from imperfection in the means of observation.

13. Describe the progress of an Insurance Company which receives, at the beginning of each year, a fixed number of new members of a given age, and in which the members only cease to be such by death.

14. Find the number of members at the end of any given number of years, and determine how long it will be before the number of deaths in a year is equal to the number of new members admitted.

15. What is the population of the United Kingdom at the present time, and what does the annual taxation amount to per head?

16. What is the effect at home and abroad of a depreciation in the value of the metallic currency of a country?

17. What consequences arise from the quantity of the coinage in circulation being excessive?

18. In what way, if at all, can the Bank of England influence the amount of circulation?

19. To what extent is the gold in the issue department of the Bank available, at any given time, in the banking department?

20. What is the precise nature of the security which a Bank of England note constitutes?

21. Describe the manner in which you would construct the accounts, or "open the books," of a Life Assurance Company commencing operations.

22. Explain the methods of determining the market value of a contingent reversion.

23. A wishes to buy an annuity for his wife, to commence at his decease; on what terms, as regards rates of interest and mortality, would he be likely to get it, and what regulates the terms? Both are in good health.

24. A has been presented with a policy of assurance on his own life, on which all the premiums have been paid; he wants to surrender this, in consideration of an annuity while he lives. In what way would you find the annuity to be given?

25. B is entitled to an estate at the death of his father, if he survive him. He has assured his life, against that of his father, for half the value of the estate, by a single payment; but he has to pay an annuity whilst he and his father are both living. Describe the way in which you would arrive at the market value of B's interest, and say what rates of interest and mortality you would use in the determination.